

武汉理工大学考试试卷 (A 卷)

2018~2019 学年 1 学期 概率论与数理统计 B 课程 闭卷

时间 120 分钟, 48 学时, 3.0 学分, 总分 100 分, 占总评成绩 70% 2019 年 1 月 4 日

题号	一	二	三	四	五	六	七	八	九	十	合计
满分	15	15	10	10	10	10	10	10	10		100
得分	9	9	10	6	5	8	2	10	10		69

$\Phi(0) = 0.5$, $\Phi(1.645) = 0.95$, $\chi_{0.025}^2(15) = 27.488$, $\chi_{0.975}^2(15) = 6.262$,
 $\chi_{0.025}^2(16) = 28.845$, $\chi_{0.975}^2(16) = 6.908$, $t_{0.025}(7) = 2.3646$, $t_{0.025}(6) = 2.4469$

得分 9

I. Fill in the blanks. (5×3')

- Suppose that $P(A) = 0.5$, $P(B) = 0.6$, $P(B|A) = 0.8$. Then $P(A \cup B) = 0.7$.
- Suppose that $X \sim N(2, \sigma^2)$, and $P\{2 < X \leq 4\} = 0.3$, then $P\{X < 0\} = 0.2$.
- Suppose that $X \sim P(\lambda)$, $\lambda > 0$, and $E[(X-2)(X-3)] = 2$, then $\lambda = 2$.
- Suppose that $X_1, X_2, \dots$ are a random sample from standard normal distribution. Let $Y = (X_1 + X_2 + X_3)^2 + (X_4 + X_5 + X_6)^2$, then determine the value of $C = \frac{1}{2}$ such that CY has a χ^2 distribution.
- Suppose that a random sample of 16 observations is taken from a normal distribution for which both the mean μ and the variance σ^2 are unknown. We got $\bar{x} = 0.081$ and $s = 6.2022$, then the 95% confidence interval for the deviation σ is $(20.991, 92.145)$.

得分 9

II. Please choose only ONE letter among A, B, C, D, and put it in the bracket within that question. (5×3')

- Let A, B be events, then we have that $P(A - B) =$
 A. $P(A) - P(B)$ B. $P(A) - P(B) + P(\bar{A}B)$
 C. $P(A) + P(AB)$ D. $P(A) + P(\bar{B}) - P(AB)$
- Given that $P(A) = 0.3$, $P(B) = 0.5$, $P(AB) = 0.15$, then
 A. $P(B|A) = P(AB)$ B. $P(B|\bar{A}) = P(BA)$
 C. $P(A|B) = P(A)$ D. $P(A|\bar{B}) = P(\bar{B})$

(B)

(C)

- If random variables X and Y has correlation coefficient $\rho = 0$, then
 A. X and Y are independent B. X and Y are not necessarily correlated
 C. X and Y are definitely uncorrelated D. X and Y are correlated
- Suppose that $X_1, X_2, \dots$ are a random sample from the population $X \sim N(\mu, \sigma^2)$, which one is the unbiased estimator of σ^2

A. $\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$

B. $\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$

C. $\frac{1}{n} \sum_{i=1}^n X_i^2$

D. $\bar{X}^2$

- Let X be a random variable and $\text{Var}(X) = \sigma^2$, then $P[|X - E(X)| < 3\sigma]$

A. $\geq \frac{8}{9}$

B. $< \frac{8}{9}$

C. $> \frac{1}{9}$

D. $\leq \frac{1}{9}$

得分 10

III. (10') A component of computer is provided by three factories A, B, and C. Suppose that 45 percent of the components are provided by factory A, 36 percent by factor B and 19 percent by factory C. Suppose further that the defective rate of the components provided by factory A, factory B and factory C is 0.05, 0.04 and 0.02 respectively.

- Find the probability that a randomly selected one component is defective.
- Suppose that one component is selected at random and it is found to be defective. Determine the probability that it is provided by factory C.

Solution (1), Supposing "a randomly selected one component is defective" is D

$$P(D) = 0.45 \times 0.05 + 0.36 \times 0.04 + 0.19 \times 0.02 = 0.0407$$

So the probability that a randomly selected one component is defective is 0.0407.

(2) Supposing "it is provided by factory C" is E .

$$P(E|D) = \frac{P(DE)}{P(D)} = \frac{0.19 \times 0.02}{0.0407} = 0.093$$

So the probability is 0.093.

得分

IV. (10') Suppose that the p.d.f. of a random variable X is as follows:

$$f(x) = Ce^{\frac{|x|}{a}}, (a > 0)$$

(1) Find the value of the constant C ; (2) Find the d.f. $F(x)$ of X ; (3) Find the value of $P(|X| < 2)$ Solution (1) $\int_{-\infty}^{+\infty} f(x) dx = 1$.

$$\textcircled{1} \int_{-\infty}^{+\infty} f(x) dx = \int_{-\infty}^0 Ce^{-\frac{x}{a}} dx + \int_0^{+\infty} Ce^{\frac{x}{a}} dx$$

$$= ac + ac = 1 \quad \text{So } C = \frac{1}{2a}$$

$$(2) f(x) = \begin{cases} \frac{1}{2a} e^{\frac{x}{a}}, & x < 0 \\ \frac{1}{2a} e^{-\frac{x}{a}}, & x \geq 0 \end{cases} \quad \text{When } x < 0, \quad \bar{F}(x) = \int_0^x \frac{1}{2a} e^{\frac{x}{a}} dx = \frac{1}{2}(e^{\frac{x}{a}} - 1)$$

$$\textcircled{2} \text{ When } x \geq 0, \quad \bar{F}(x) = \int_0^x \frac{1}{2a} e^{-\frac{x}{a}} dx = \frac{1}{2}(1 - e^{-\frac{x}{a}})$$

$$\bar{F}(x) = \begin{cases} \frac{1}{2} e^{\frac{x}{a}} - \frac{1}{2}, & x < 0 \\ \frac{1}{2} - \frac{1}{2} e^{-\frac{x}{a}}, & x \geq 0 \end{cases}$$

$$(3) P(|X| < 2) = P(-2 < X < 2) = \bar{F}(2) - \bar{F}(-2) = 1 - e^{-\frac{2}{a}}$$

得分

V. (10') Suppose that X has an exponential distribution with parameter 2, which denoted by $X \sim E(2)$. Determine the p.d.f. of $Y = 1 - e^{-2X}$.

$$\text{Solution } f_X(x) = e^{-2x}, \quad x \geq 0$$

$$F_Y(y) = P(Y \leq y) = P(1 - e^{-2X} \leq y) = P(X \leq \frac{\ln(1-y)}{-2})$$

$$= \int_0^{\frac{\ln(1-y)}{-2}} e^{-2x} dx = \frac{1}{2}(e^{-\ln(1-y)} - 1)$$

$$f_Y(y) = F'_Y(y) = \begin{cases} \frac{1}{2(1-y)} e^{-\ln(1-y)}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

得分

VI. (10') Suppose that the joint p.d.f. of (X, Y) is:

$$f(x, y) = \begin{cases} \frac{1}{2}(x+y)e^{-(x+y)}, & x > 0, y > 0, \\ 0, & \text{otherwise} \end{cases}$$

1) Determine whether or not X and Y are independent and give your reason.2) Determine the p.d.f. of $Z = X + Y$.

$$\text{Solution (1), } f_X(x) = \int_{-\infty}^{+\infty} f(x, y) dy = \int_0^{+\infty} \frac{1}{2}(x+y)e^{-(x+y)} dy$$

$$\textcircled{1} x > 0 \quad = -\frac{(x+y+1)}{2} e^{-(x+y)} \Big|_0^{+\infty} = \frac{x+1}{2} e^{-x}$$

$$y > 0, \quad f_Y(y) = \int_{-\infty}^{+\infty} f(x, y) dx = \frac{y+1}{2} e^{-y}$$

$$f_X(x) = \begin{cases} \frac{x+1}{2} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}, \quad f_Y(y) = \begin{cases} \frac{y+1}{2} e^{-y}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

Since $f_X(x) \cdot f_Y(y) \neq f(x, y)$, X and Y are not independent.

$$(2) Z = X + Y, \quad Y = Z - X$$

$$\text{If } Z \leq 0, \quad f_Z(Z) = 0$$

$$\text{If } Z > 0, \quad f_Z(Z) = \int_0^Z f(x, Z-x) dx$$

$$= \int_0^Z \frac{1}{2}(x+1)e^{-x} \times \frac{Z-x+1}{2} e^{-(Z-x)} dx$$

$$= \left(\frac{1}{6} Z^3 + Z^2 + Z \right) \times \frac{e^{-Z}}{4}$$

$$\text{So } f_Z(Z) = \begin{cases} \left(\frac{1}{24} Z^3 + \frac{1}{4} Z^2 + \frac{1}{4} Z \right) e^{-Z}, & Z > 0 \\ 0, & \text{otherwise} \end{cases}$$

得分

2

VII. (10') Suppose that a machine produces a nondefective item with probability 0.8. What is the smallest number of items that have to be produced in order for the probability to be at least 0.9 that the proportion of nondefective in this batch of items will be between 76% and 84%? (Please use the central limit theorem to give the estimation)

Solution

~~Let x be the probability.~~

Let n be the number of items.

$$P\{76\% \leq 0.8x \leq 84\%\} > 0.9$$

$$\frac{0.84n - 0.8n}{0.8} = \frac{0.8 - 0.76}{0.8} = \Phi(1.645) - \Phi(0) = 0.95$$

$$\frac{0.9n - 0.8n}{\sqrt{n \times 0.8 \times 0.2}} = 1.645 \quad n = 6$$

得分

10

VIII. (10') Suppose that $X_1, X_2, \dots, X_n$ are a random sample from population X for which the p.d.f. is

$$f(x; \theta) = \begin{cases} \frac{\theta^2}{x^3} e^{-\frac{\theta}{x}}, & \text{for } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where parameter $\theta > 0$ is unknown and that $x_1, x_2, \dots, x_n$ are observed values of $X_1, X_2, \dots, X_n$.

(1) Determine the moment estimators of θ .

(2) Determine the maximum likelihood estimator of θ .

Solution 1, $E(X) = \int_0^{+\infty} x f(x, \theta) dx = \theta, \quad x > 0.$

$\bar{X} = \hat{\theta}$. So the moment estimators of θ is $\bar{X}$.

$$(2) L(\theta) = \left(\frac{\theta^2}{x^3}\right)^n \prod_{i=1}^n e^{-\frac{\theta}{x_i}}$$

$$\ln L(\theta) = n \ln \frac{\theta^2}{x^3} - \sum_{i=1}^n \frac{\theta}{x_i}$$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = 0 = \frac{2n}{\theta} - \sum_{i=1}^n \frac{1}{x_i} \quad \text{So } \hat{\theta} = \frac{2n}{\sum_{i=1}^n \frac{1}{x_i}}$$

得分

10

IX. (10') Suppose that lifetime, in hours, of an electric component have a normal distribution. Test the hypothesis that the average lifetime of an electric component is 23.8 hours if the lifetimes of a random sample of 7 components are 26.7, 22.0, 24.1, 21.0, 27.2, 25.0 and 23.4 hours. Use a 0.05 level of significance to confirm that the distribution of the lifetime is normal.

Solution $H_0: \mu = \mu_0 = 23.8, \quad H_1: \mu \neq \mu_0$

$$\alpha = 0.05$$

$$T = \left| \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \right| \sim$$

$$n=7, \quad \bar{X} = \frac{26.7+22+24.1+21+27.2+25+23.4}{7} = 24.2$$

$$S = \sqrt{\frac{1}{n-1} [(24.2-26.7)^2 + (24.2-22)^2 + (24.2-24.1)^2 + (24.2-21)^2 + (24.2-27.2)^2 + (24.2-25)^2 + (24.2-23.4)^2]} = 2.3$$

$$t = \left| \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \right| = \left| \frac{24.2 - 23.8}{2.3/\sqrt{7}} \right| = 0.46$$

$$t_{\alpha/2}(n-1) = t_{0.025}(6) = 2.4469$$

$$t < t_{\alpha/2}(6)$$

So the distribution of the lifetime is normal.